Corners and other hereditary subalgebras of graph C^*-algebras

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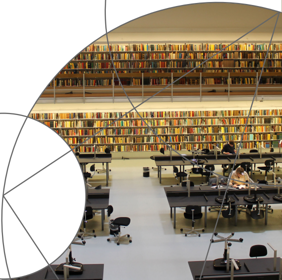

What is a graph C^*-algebra?

Definition

Let $E = (E^0, E^1, r, s)$ be a (countable, directed) graph. Its graph C^*-algebra $C^*(E)$ is the universal C^*-algebra generated by mutually orthogonal projections $\{p_v \mid v \in E^0\}$ and partial isometries $\{s_e \mid e \in E^1\}$ satisfying the relations

1. $s_e^* s_f = 0$ if $e, f \in E^1$ and $e \neq f$,
2. $s_e^* s_e = p_{r(e)}$ for all $e \in E^1$,
3. $s_e s_e^* \leq p_{s(e)}$ for all $e \in E^1$,
4. $p_v = \sum_{e \in s^{-1}(v)} s_e s_e^*$ for all $v \in E^0$ with $0 < |s^{-1}(v)| < \infty$.

Proposition (A-Ruiz)

Let E be a graph. Then the following are equivalent:

1. E is finite graph with no sinks.
2. $C^*(E)$ is a Cuntz-Krieger algebra.
3. $C^*(E)$ is unital and

$$\operatorname{rank}(K_0(C^*(E))) = \operatorname{rank}(K_1(C^*(E))).$$

Corners of Cuntz-Krieger algebras

Theorem (A-Ruiz)

Let $\mathfrak{A}$ be a Cuntz-Krieger algebra.

1. *If p is a nonzero projection in $\mathfrak{A} \otimes \mathbb{K}$, then $p(\mathfrak{A} \otimes \mathbb{K})p$ is a Cuntz-Krieger algebra.*
2. *If p is a nonzero projection in $\mathfrak{A}$, then $p\mathfrak{A}p$ is a Cuntz-Krieger algebra.*

Corollary

Let $\mathfrak{B}$ be a unital C^*-algebra. If $\mathfrak{B}$ is stably isomorphic to a Cuntz-Krieger algebra, then $\mathfrak{B}$ is a Cuntz-Krieger algebra.

Theorem (Ara-Moreno-Pardo)

Let E be a row-finite graph. Any projection in $C^*(E) \otimes \mathbb{K}$ is Murray-von Neumann equivalent to a projection of the form

$$\sum_{v \in E^0} m_v p_v$$

with all but finitely many m_v equal to zero.

Nonstable K-theory

Theorem (Ara-Moreno-Pardo)

Let E be a row-finite graph. Any projection in $C^*(E) \otimes \mathbb{K}$ is Murray-von Neumann equivalent to a projection of the form

$$\sum_{v \in S} m_v p_v$$

where $m_v \geq 1$ for all $v \in S$, and S is a finite subset of E^0.

Theorem (Hay-Loving-Montgomery-Ruiz-Todd)

Let E be a graph. Any projection in $C^*(E) \otimes \mathbb{K}$ is Murray-von Neumann equivalent to a projection of the form

$$\sum_{(v,T) \in S} n_{(v,T)} \left(p_v - \sum_{e \in T} s_e s_e^* \right)$$

where $m_{(v,T)} \geq 1$ for all $(v, T) \in S$, and S is a finite subset of

$$\{(v, T) \in E^0 \times 2^{E^1} \mid T \subseteq s_E^{-1}(v), T \text{ is finite}, T = \emptyset \text{ when } |s_E^{-1}(v)| < \infty\}.$$

Hereditary subalgebras of graph C^*-algebras

Theorem (A-Gabe-Ruiz)

Let E be a graph with finitely many vertices.

1. *Let $\mathfrak{A}$ be a hereditary subalgebra of $C^*(E) \otimes \mathbb{K}$. If $\mathfrak{A}$ is σ_p-unital, then $\mathfrak{A}$ is a graph C^*-algebra.*
2. *Let $\mathfrak{A}$ be a hereditary subalgebra of $C^*(E)$. If $\mathfrak{A}$ is σ_p-unital, then $\mathfrak{A}$ is a graph C^*-algebra.*

Corollary

Let $\mathfrak{A}$ be a σ_p-unital C^*-algebra, and let E be a graph with finitely many vertices. If $\mathfrak{A}$ is stably isomorphic to $C^*(E)$, then $\mathfrak{A}$ is a graph C^*-algebra.